Math 301 Final Exam Practice Problems

1. Your answer to the following questions does not need to contain any explanation. If a number is naturally written using binomials, factorials, or powers, then please write it that way.

   Practice note: You should explain every part of your answer as best you can.

   (a) What is the sum of the coefficients in the expansion of $(x+y+z)^{16}$?

   (b) A 2-colorable graph has 18 vertices. What is the largest number of edges the graph can have?

   (c) What is the coefficient of x^7 in the expansion of $(2+x)^{26}$?

   (d) How large does N have to be to guarantee that any list of N integers contains at least 4 integers which have the same remainder when divided by 19?

   (e) How many subsets with 5 elements does a set with n elements have? Your answer should be in terms of n.

   (f) Use that $16 \times 10 = 53 \times 3 + 1$ to solve for $0 \leq x < 53$ satisfying:

   $$16x \equiv 6 \mod 53$$

   (g) How many ways are there to distribute 7 identical nickels and 23 identical quarters to 6 of your friends if everyone has to get at least 25 cents?

   (h) How many ways are there to distribute 200 identical nickels and 40 identical quarters to 6 of your friends if everyone has to get exactly 75 cents?

   (i) A planar map has 10 vertices and 20 edges. How many faces does it have?

   (j) How many anagrams are there of the word ASSASSINATION so that no two vowels are next to each other?

   (k) How many ways are there to seat (indistinguishable) people in a row of 6 chairs so that no two people have to sit next to each other?

   (l) How many ways are there to tile a $2 \times n$ chessboard with 2×1 dominoes? Hint: Use a recurrence relation.

   (m) How many edges does a tree with 14 vertices have?

2. Use induction to prove that $3n + 2 \leq n^2$ for all integers $n \geq 4$.

3. Let F_n denote the nth Fibonacci number, starting with $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, and then defined by the recurrence $F_k = F_{k-1} + F_{k-2}$.

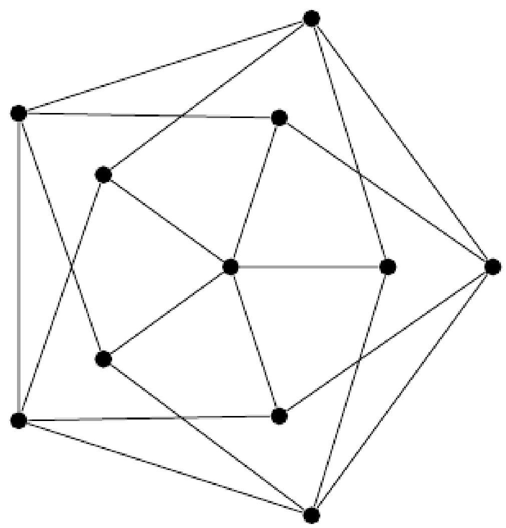
   Use induction to prove that $F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$.

4. (a) Draw the tree T corresponding to the Prüfer code 2942701234.

   (b) How many edges does T have?

   (c) How many faces are there in any planar drawing of the tree T?

5. The following questions are about the graph G drawn below, which has 11 vertices and 20 edges.



(a) Is G 2-colorable? Explain.

(b) For which number k does Brooks's theorem guarantee: You can definitely color G with k-colors.

(c) Show by example that you can 4-color G.

(d) Draw a spanning tree of G.

(e) Does G have any Eulerian walks? Explain.

(f) G has a Hamiltonian cycle. Can you find it?