Math 317 Midterm Exam #1 Practice Problems

1. Give an example for each of the following, or explain conclusively and clearly why one cannot exist, stating any facts, definitions, and theorems that apply.
   (a) An alternating sequence which converges. (A sequence is *alternating* if its terms alternate between positive and negative)
   (b) A sequence with no convergent subsequence.
   (c) A decreasing sequence which diverges.
   (d) A sequence (s_n) with $\liminf s_n = -\infty$ and $\limsup s_n = +\infty$.

2. Answer whether each statement is true or false. If the statement is true, give a brief explanation. If the statement is false, provide a counterexample.
   (a) Every bounded sequence contains its supremum.
   (b) Every Cauchy sequence is bounded.
   (c) Every increasing sequence is bounded.
   (d) If a sequence converges to a number L, then so do all subsequences.

3. Find the limit of the sequence $\left(\frac{2n-3}{3n+1}\right)$ and prove that your limit is correct.

4. Prove that the sequence (s_n) defined by $s_1 = 1$ and $s_{n+1} = \sqrt{3 + s_n}$ does **not** converge to 4.

5. A sequence is *periodic* if there exists p so that $a_{n+p} = a_n$ for all natural numbers n. Let a_n be a periodic sequence for which $a_1 \neq a_{2019}$. Prove that (a_n) does not converge. (Hint: Is it Cauchy?)